

Transverse-momentum-dependent parton distributions (TMDs)

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Abstract. Transverse-momentum-dependent parton distributions (TMDs) provide three-dimensional images of the partonic structure of the nucleon in momentum space. We made impressive progress in understanding TMDs, both from the theoretical and experimental point of view. This brief overview on TMDs is divided in two parts: in the first, an essential list of achievements is presented. In the second, a selection of open questions is discussed.

Keywords: parton distribution functions, semi-inclusive DIS, transverse momentum

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Providing detailed maps of the nucleon partonic structure is one of the main goals of subnuclear physics. Such maps are the starting steps toward the comprehension of confined QCD dynamics. They are essential for the interpretation of any high-energy process involving hadrons.

After about forty years of study, we have reached a good knowledge of (some) collinear parton distribution functions (PDFs). They map the partonic structure in a monodimensional momentum space, i.e., as a function of partonic longitudinal momentum. “Longitudinal” is defined as parallel to the hard probe, which must be always present in hard processes where PDFs are observed (e.g., in DIS the photon with virtuality Q^2).

TMDs (an acronym for Transverse Momentum Distributions or Transverse Momentum Dependent parton distribution functions) are natural extensions of standard PDFs: they are three-dimensional maps of the partonic structure in momentum space. They depend not only on the longitudinal momentum, but also on the transverse momentum.

Ultimately, the knowledge of TMDs will allow us to build tomographic images of the inner structure of the nucleon in momentum space, complementary to the impact-parameter space tomography that can be achieved by studying generalized parton distribution functions (GPDs). An example of tomographic images of the nucleon based on a model calculation of TMDs [1] is presented in Fig. 1.

The information contained in TMDs is as fundamental as that contained in standard PDFs. They reveal crucial aspects of the dynamics of confined partons, they can be extracted from experimental data, and allow us to make prediction for hard-scattering experiments involving nucleons.

Our understanding of TMDs and their extraction from data has made giant steps in the last years, thanks to new theoretical ideas and experimental measurements (see Ref. [2] for a recent review). In the near future, more experimental data are expected from HERMES, COMPASS, BELLE and JLab. A high-energy, high-luminosity, polarized electron-proton collider, such as the envisaged EIC project, would undoubtedly be a precision machine for the study of TMDs.

WHAT WE KNOW ABOUT TMDS

Similarly to standard collinear PDFs, it is essential to define TMDs in a formally clear way, through the proof of factorization theorems. TMDs appear when factorizing semi-inclusive processes. For instance, while totally inclusive DIS can be described introducing collinear PDFs, TMDs appear in semi-inclusive DIS if the transverse momentum of one outgoing hadron, $P_{h\perp}$, is measured.

Dealing with semi-inclusive processes pushes the difficulty of proving factorization theorems to a higher level of complications. TMD factorization is in fact a challenging arena where many of the simplifications used in collinear factorization cannot be applied. Nevertheless, factorization for semi-inclusive DIS has been worked out explicitly at leading twist (twist 2) and one-loop order [3]. For instance, the structure function $F_{UU,T}$ in the region $P_{h\perp}^2 \ll Q^2$ can

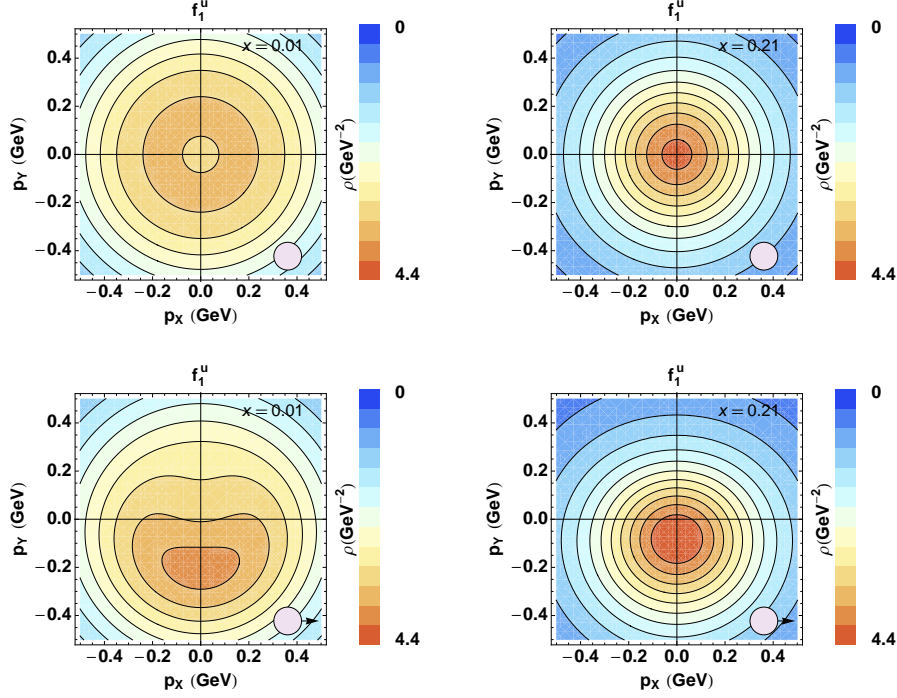


FIGURE 1. Momentum-space tomographic “images” of the up quarks in a nucleon obtained from a model calculation of TMDs [1]. The nucleon is assumed to move into the page. The circle with the arrow indicates the nucleon and its spin orientation. The distortion in the lower panels is due to the Sivers function.

be expressed as

$$F_{UU,T} = |H(x\zeta^{1/2}, z^{-1}\zeta_h^{1/2}, \mu_F)|^2 \sum_a x e_a^2 \int d^2 p_T d^2 k_T d^2 l_T \times \delta^{(2)}(p_T - k_T + l_T - P_{h\perp}/z) f_1^a(x, p_T^2; \zeta, \mu_F) D_1^a(z, k_T^2; \zeta_h, \mu_F) U(l_T^2; \mu_F). \quad (1)$$

The formula contains the (calculable) hard scattering factor H , the transverse-momentum-dependent PDFs and fragmentation functions, and the soft factor U , a nonperturbative, process-independent object. However, it should be also possible to include the soft factor into the PDFs and fragmentation functions by redefining them in a suitable way [4].

A schematic definition of quark TMDs is (taking, e.g., the unpolarized distribution of a quark with flavor a)

$$f_1^a(x, p_T^2; \zeta, \mu_F) = \int \frac{d\xi^- d^2 \xi_T}{(2\pi)^3} e^{ip \cdot \xi} \langle P | \bar{\psi}^a(0) \mathcal{L}_{(\pm\infty, 0)}^{v\dagger} \gamma^+ \mathcal{L}_{(\pm\infty, \xi)}^v \psi^a(\xi) | P \rangle \Big|_{\xi^+ = 0}. \quad (2)$$

The Wilson lines, \mathcal{L} , guarantee the gauge invariance of the TMDs. They depend on the gauge vector v and contain also components at infinity running in the transverse direction [5]. Beyond tree level, there are several subtleties related to the treatment of soft and rapidity divergences. Various ways to deal with these divergences have been proposed [3, 6]. In general, they require the introduction of a rapidity cutoff ζ , on which the TMDs depend.

A remarkable property of TMDs is that the detailed shape of the Wilson line is process-dependent. This immediately leads to the conclusion that TMDs are not universal. However, for transverse-momentum-dependent fragmentation functions, the shape of the Wilson line appears to have no influence on physical observables [7]. In SIDIS and Drell–Yan, the difference between the Wilson line consists in a simple direction reversal and leads to calculable effects, namely a simple sign reversal of all T-odd TMDs [8].

In general and in simplified terms, the study of TMD factorization requires a deeper understanding of what happens when a quark is hit inside a nucleon, with a particular attention to the infinitely many gluons that surround the quark, beyond the simple case when its transverse momentum is integrated over.

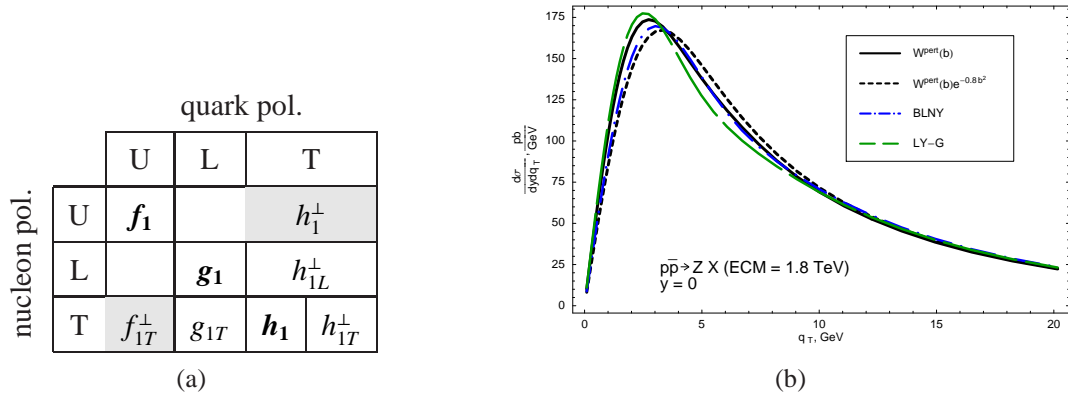


FIGURE 2. (a) Twist-2 transverse-momentum-dependent distribution functions. The U,L,T correspond to unpolarized, longitudinally polarized and transversely polarized nucleons (rows) and quarks (columns). Functions in boldface survive transverse momentum integration. Functions in gray cells are T-odd. (b) The cross sections of Z boson production at the Tevatron, computed in the CSS formalism. The difference between the curves shows the impact of choosing different nonperturbative components for the TMDs. See Ref. [12] for details.

At present, especially for azimuthally-dependent structure functions, phenomenological analyses are often carried out using the tree-level approximated expression (which corresponds to using parton-model formulas for collinear PDFs and neglecting scaling violations).

$$F_{UU,T} = \sum_a x e_a^2 \int d^2 p_T d^2 k_T \delta^{(2)}(p_T - k_T - P_{h\perp}/z) f_1^a(x, p_T^2) D_1^a(z, k_T^2). \quad (3)$$

The transverse-momentum dependence of the partonic functions is usually assumed to be a flavor-independent Gaussian [9]. The tree-level approximation and the Gaussian assumption are known to be inadequate at $P_{h\perp}^2 \gg M^2$, but they could effectively describe the physics at $P_{h\perp}^2 \approx M^2$. Especially for low-energy experiments, this is where the bulk of the data is.

There is an extensive literature where the analysis is carried out to a higher level of complication, but only for the specific case of unpolarized observables integrated over the azimuthal angle of the measured transverse momentum. The analysis is usually performed in the space of the Fourier-conjugate to $P_{h\perp}$ (b -space) in the Collins–Soper–Sterman (CSS) framework [10]. The region of $P_{h\perp}^2 \gg M^2$, or $b^2 \ll 1/M^2$, can be calculated perturbatively, but when $P_{h\perp}^2 \approx M^2$ a nonperturbative component has to be introduced and its parameters must be fitted to experimental data. This component is usually assumed to be a flavor-independent Gaussian [11].

At present, we can make the conservative statement that unpolarized quark TMDs seems to be well described by flavor-independent Gaussians with $\sqrt{\langle p_T^2 \rangle} \approx 0.4 - 0.8$ GeV, depending on the kinematics.

The knowledge of the details of the unpolarized TMDs has an impact also on high-energy physics. In Fig. 2b, the cross section for Z boson production at the Tevatron is plotted [12]. The difference between the curves originates from different models and fits of the nonperturbative component of the TMDs. Apart from the details, the plot shows that the knowledge of TMDs is essential for precision studies at the Tevatron.

Even the determination of a fundamental parameter of the Standard Model, the mass of the W boson, is affected by the uncertainties of the knowledge of unpolarized TMDs. In Ref. [13], the CDF collaboration discussed several ways to fit the W mass. According to the analysis, TMDs uncertainties generate an error of 3.9 MeV on the W mass determination (the total systematic error is about 34 MeV).

When the spin of the nucleon and that of the quark are taken into account, eight twist-2 TMDs can be introduced. They are listed in Fig. 2a. As with collinear PDFs, extracting TMDs calls for global fits to semi-inclusive DIS, Drell–Yan, and e^+e^- -annihilation data.

From the theoretical point of view, we know positivity bounds for TMDs [14], behavior at high transverse momentum [15], behavior at high x [16].

Apart from the f_1 (unpolarized function) and the p_T -integral of g_1 (helicity distribution), we have at the moment some extractions of h_1 (transversity distribution), f_{1T}^\perp (Sivers function) [17, 18] and h_1^\perp (Boer–Mulders function) [19].

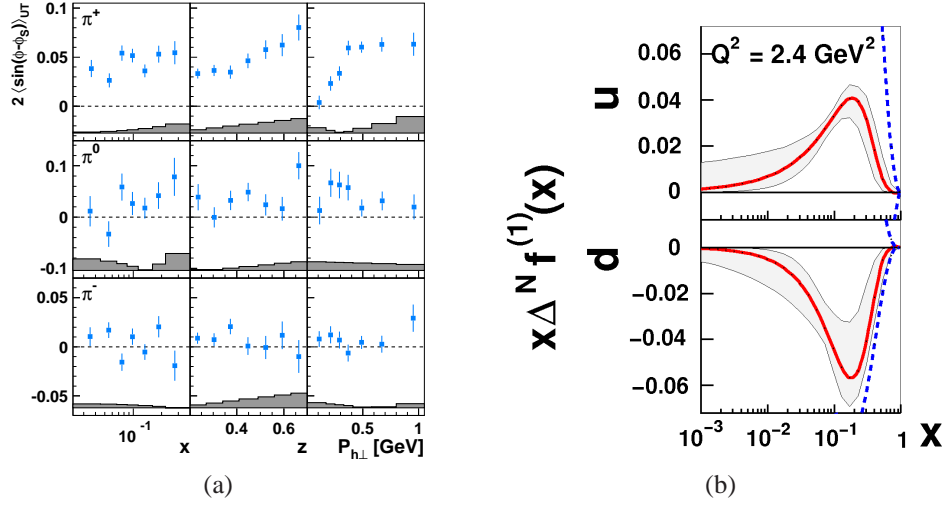


FIGURE 3. (a) Example of experimental data on the Siverts asymmetries [20]. (b) Siverts distribution for up and down quark from a recent extraction [17].

As illustrated in Fig. 1, a nonzero Siverts function means that the distribution of quarks in transverse momentum is affected by the direction of the nucleon’s spin. Thanks to the experimental measurements of the HERMES [20] and COMPASS [21] collaborations, it has become possible to extract for the first time the Siverts distribution function. Fig. 3 shows a representative subset of the data and one of the most recent parametrizations [17, 18].

The conclusions we can draw from these first studies are already interesting: the Siverts function is nonzero, at least for up and down quarks; the Siverts function for up quark is negative, while for down quark is positive and could be even larger than that of up quarks. These findings are in agreement with the idea suggested in Ref. [22] of a link between the anomalous magnetic moment, κ , and the Siverts function. Assuming isospin symmetry and no strange quark contribution, we know that $\kappa_u \approx 1.673$ and $\kappa_d \approx -2.033$. Since quarks are subject to attractive final-state color interactions, this indeed translates in a negative up and a large, positive down Siverts, in qualitative agreement with some models [23].

WHAT WE STILL DON’T KNOW ABOUT TMDs

There are many essential questions that have not been addressed sufficiently in the past.

Concerning the formalism, there are a few alternative definitions of TMDs, based on different ways to handle soft and rapidity divergences. It is at present not clear if the different definitions are all formally acceptable, if they can be seen as different “schemes” to define TMDs, and if some of them are more convenient than others.

It is extremely important to clarify if and how TMDs can be studied in hadron-hadron collisions to hadrons. The treatment of Wilson lines in this case is more intricate and recent studies lead to the conclusion that TMD factorization does not work [24]. Efforts should be made to look for experimental evidence of factorization-breaking effects, and to find ways to avoid problems [].

For what concerns our phenomenological knowledge of TMDs, we do not know whether the transverse momentum distribution is different for different quark flavors and for gluons. We know that collinear PDFs are flavor dependent: not only their normalization is different, but also their shape. Their transverse spatial distributions, as reconstructed from form factor measurements, are also sharply different (see, e.g., [25]). Some models predict different shapes [1, 26]. There is no fundamental reason to believe that their transverse momentum distributions should be equal. However, up to now they have been assumed to be exactly the same in all phenomenological analyses. There are in fact already some indications that this might not be a good assumption, both from experiments [27] and lattice-QCD calculations [28] (see Fig. 4a).

We don’t know enough about the detailed shape of TMDs. In all studies, it is assumed to be Gaussian. However, this is a choice based on convenience and tradition, but not on a fundamental reason. There is no model calculation of TMDs that predicts a pure Gaussian shape. The presence of nonzero orbital angular momentum actually implies that

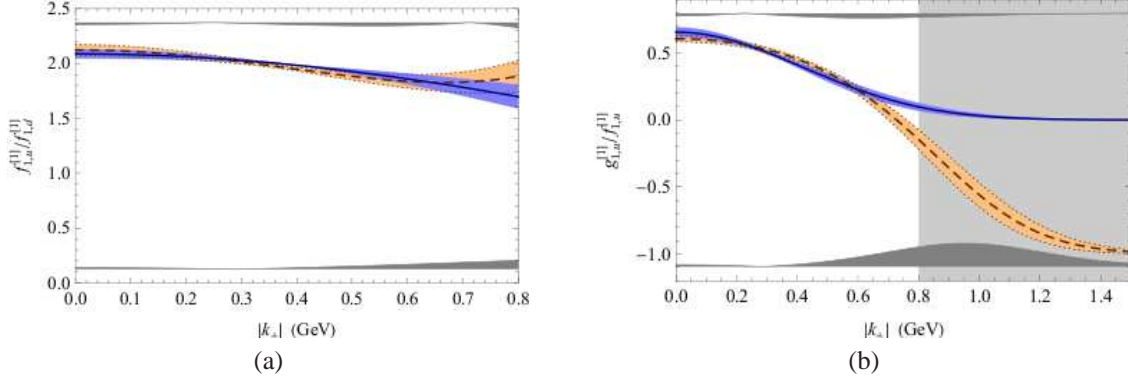


FIGURE 4. First hints on TMD behavior based on lattice QCD computations [28]. (a) Ratio between the TMD of up and down quarks from two different fits to lattice QCD data: the non-constant ratio suggests that up and down quark have different transverse momentum distributions. (b) Ratio between the helicity distribution and the unpolarized distribution for up quarks: the non-constant ratio suggests that quarks with different spin orientation have different transverse momentum distributions.

the shape cannot be a simple Gaussian. In nonrelativistic quantum mechanics, it is well known that wave-functions with orbital angular momentum vanish at zero momentum. This feature is reflected also in TMDs: contributions from partons with nonzero angular momentum have to vanish at zero transverse momentum and therefore cannot be described by a simple Gaussian. We can assume that the wave-function of the proton contains quarks with $L_z = 0$ (s wave) and $L_z = 1$ (p wave), where L_z is defined in the Jaffe–Manohar way. Then we know that

$$\lim_{p_T \rightarrow 0} |\psi_{p\text{-wave}}|^2 \propto p_T^2/M^4 \quad (4)$$

The full TMD will be the sum of the squares of different components of the wave-function, making it difficult to identify the contributions with nonzero L . In any case, a downturn of a TMD at small transverse momentum may signal the presence of nonzero orbital angular momentum. While this effects could barely be visible in unpolarized TMDs, certain combinations of polarized TMDs could filter out more clearly the configurations with nonzero orbital angular momentum.

Apart from the details of their shape, all the TMDs that are not boldface in Fig. 2a vanish in the absence of orbital angular momentum due to angular momentum conservation. Measuring any one of them to be nonzero is already a indisputable indication of the presence of partonic orbital angular momentum. We know already from other sources (e.g., the measurement of nucleons’ anomalous magnetic moments) that partonic orbital angular momentum is not zero, however TMDs have the advantage that they can be flavor-separated and that they are x dependent. Thus, they allow us to say if orbital angular momentum is present for each quark flavor and for gluons, and at each value of x .

If stating that a fraction of partons have nonzero orbital angular momentum is relatively simple, it is not easy to make a quantitative estimate of the net partonic orbital angular momentum using TMDs. Any statement in this direction is bound to be model-dependent. Generally speaking, TMDs have to be computed in a model and the parameters of the model have to be fixed to reproduce the TMDs extracted from data. Then, the total orbital angular momentum can be computed in the model. Unfortunately, it is possible that two models describe the data equally well, but give two different values for the total orbital angular momentum.

Another aspect that requires further investigations is to which extent transverse momentum distribution is influenced by the spin of the quarks and of the nucleon. For instance, it may be possible that the average transverse momentum of quarks with spin antiparallel to the nucleon is larger than that of quarks with spin parallel to the nucleon. Also in this case there is already some evidence from experiments [29] (see Fig. 5) and lattice-QCD calculations [28] (see Fig. 4b).

Finally, it is worth mentioning that during this conference the first measurement connected to the so-called “worm-gear” TMD g_{1T} has been reported [30] and seems to confirm the sign of the distribution predicted by lattice QCD studies [31].

In summary, TMDs open new dimensions in the exploration of the partonic structure of the nucleon. They require challenging extensions of the standard formalism used for collinear parton distribution functions, leading us to a deeper understanding of QCD. Among other things, they give evidence of the presence of partonic orbital angular momentum and, with model assumptions, they can help constraining its size.

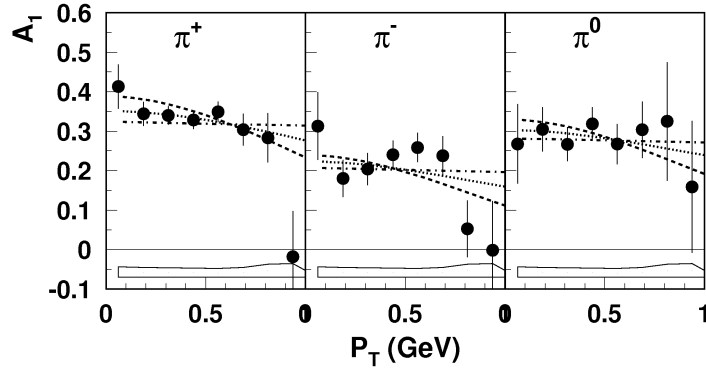


FIGURE 5. Data from the CLAS collaboration showing the transverse-momentum dependence of the double longitudinal spin asymmetry A_1 [29]. The dependence would be flat if TMDs for quarks with opposite helicity were the same.

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